A GENERALIZED DESIGN METHOD FOR DIRECTIVITY PATTERNS OF SPHERICAL MICROPHONE ARRAYS

ENZO DE SENA, HUSEYIN HACIHABIBOGLU, ZORAN CVETKOVIC
KING'S COLLEGE LONDON, CENTRE FOR TELECOMMUNICATIONS RESEARCH
METU, INFORMATICS INSTITUTE

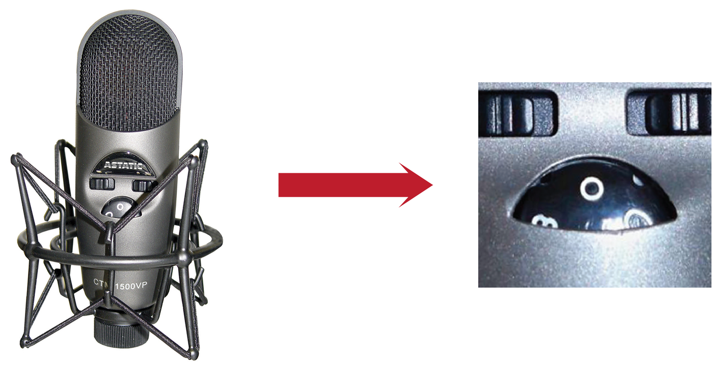
PROBLEM

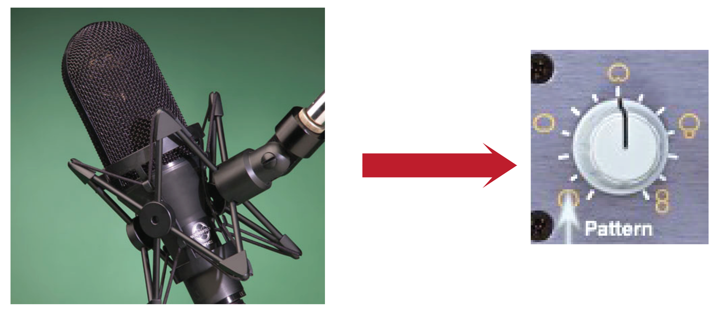
- Sound engineers and technicians usually choose the position of the microphones and their directivity pattern heuristically and according to their personal taste.
- Most commercial microphones have directivity patterns:

$$\Gamma(\theta) = 1 - a_1 + a_1 \cos\theta$$

Pattern	a_1	Description
Omnidirectional	0	Non-selective.
Cardioid	1/2	Most widely used. Good rejection for back directions.
Subcardioid	0.3	Slightly selective. Used in classical music recordings.
Supercardioid	0.586	The one with maximum front/back energy ratio.
Hypercardioid	2/3	The one with maximum directivity index.

- Variable-pattern microphones can have all values $a_1 \in (0,1)$, hence can emulate all the above and also useful hybrids between them.

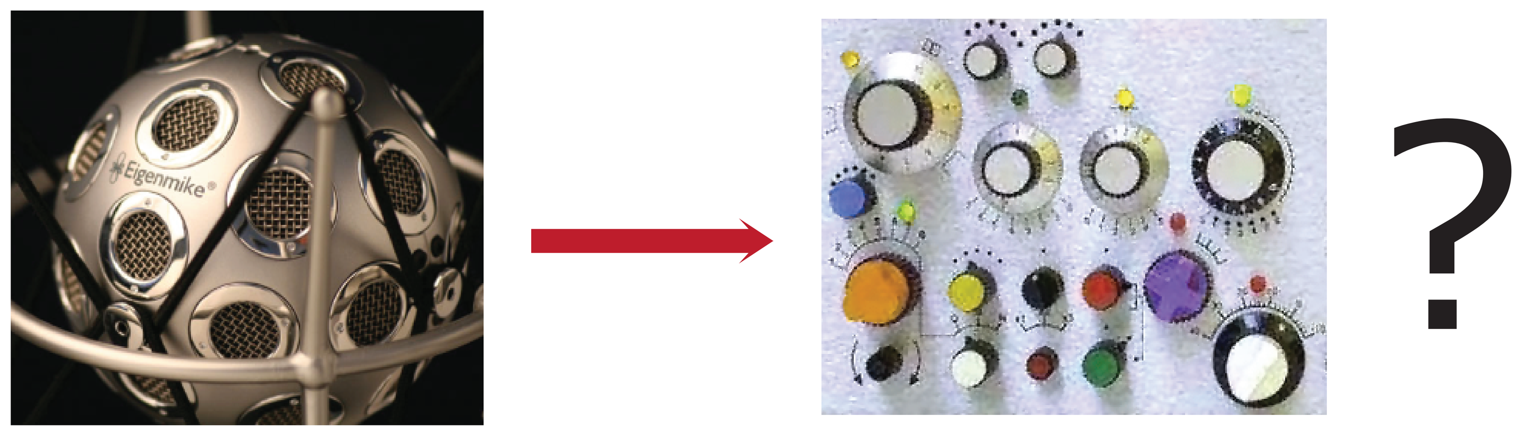




- Phase-mode beamformers like the eigenmike, and (more in general) N-order microphones greatly enlarge the design space, and can emulate patterns of the kind:

$$\Gamma_{\mathbf{a}}(\theta) = a_0 + a_1 \cos\theta + a_2 \cos^2\theta + \ldots + a_N \cos^N \theta$$

- **How can sound engineers and technicians exploit this flexibility?**
- Coefficients of higher-order cardioid, supercardioid and hypercardioid are already known, but this set of patterns is limited.
- Setting individually the N dials (coefficients) is complicated because each one of them does not have a clear impact on the shape of the directivity:



DESIGN METHOD

- Select the coefficients $\mathbf{a} = [a_0, a_1, \ldots, a_N]$ as the solution of:

$$\tilde{\mathbf{a}}(\alpha,\lambda) = \underset{\mathbf{a}}{\operatorname{argmin}}\ \Phi_{\mathbf{a}}(\alpha,\lambda) \quad \text{subject to } \Gamma_{\mathbf{a}}(\theta) \le 1 \forall\theta$$

$$\Phi_{\mathbf{a}}(\alpha,\lambda) = \lambda \frac{\int_\alpha^\pi |\Gamma_{\mathbf{a}}(\theta)|^2 \sin\theta\, d\theta}{\int_0^\alpha |\Gamma_{\mathbf{a}}(\theta)|^2 \sin\theta\, d\theta} + (1-\lambda)\int_0^\alpha |\Gamma'_{\mathbf{a}}(\theta)|^2 \sin\theta\, d\theta$$

- Angle α is set to cover the angular region where the sound sources are located, while λ controls the relative importance of the uniformity of the directivity in the desired region (such that sources are recorded at the same level) and the suppression of sources outside of it.
- Most common patterns are solutions with specific couples of (α,λ).
- More general solutions can be obtained by freely varying (α,λ).

Omnidirectional

$$\Phi_{\mathbf{a}}(\pi,0) = \int_0^\pi |\Gamma'_{\mathbf{a}}(\theta)|^2 \sin\theta\, d\theta$$

whose solution is the constant function, i.e. omnidirectional.

Supercardioid

$$\Phi_{\mathbf{a}}\left(\frac{\pi}{2},1\right) = \frac{\int_{\frac{\pi}{2}}^\pi |\Gamma_{\mathbf{a}}(\theta)|^2 \sin\theta\, d\theta}{\int_0^{\frac{\pi}{2}} |\Gamma_{\mathbf{a}}(\theta)|^2 \sin\theta\, d\theta}$$

which is the back/front ratio, hence the solution is the supercardioid.

Hypercardioid

$$\tilde{\mathbf{a}}(0,1) = \underset{\mathbf{a}}{\operatorname{argmin}} \int_0^\pi |\Gamma_{\mathbf{a}}(\theta)|^2 \sin\theta\, d\theta$$

which is the inverse of DI, hence the solution is the hypercardioid.

Cardioid and subcardioid

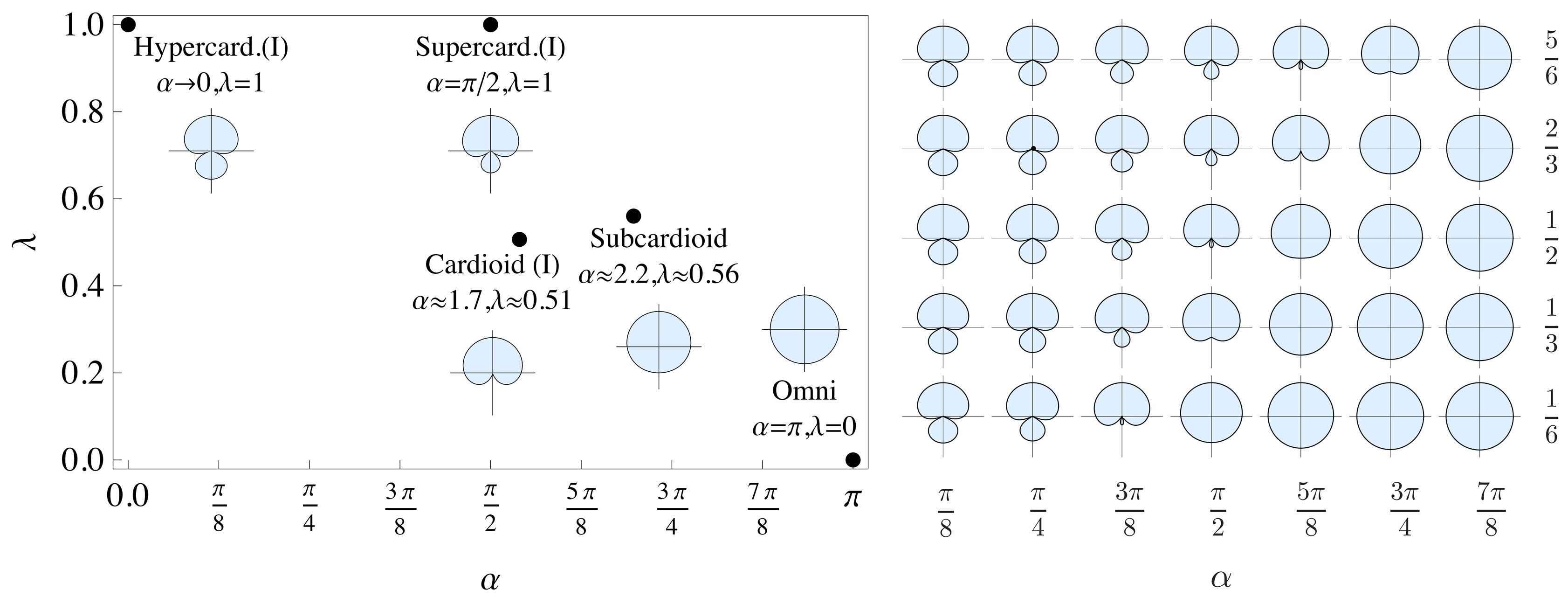
- Not originally designed to satisfy any specific optimal criteria, and are not particular cases of the proposed cost function.
- Very close to solutions of the optimization problem.

$$\delta = \min_{\alpha,\lambda} \frac{1}{2\pi}\int_0^{2\pi} \left|\Gamma_{\tilde{\mathbf{a}}(\alpha,\lambda)}(\theta) - \Gamma_d(\theta)\right|^2 d\theta$$

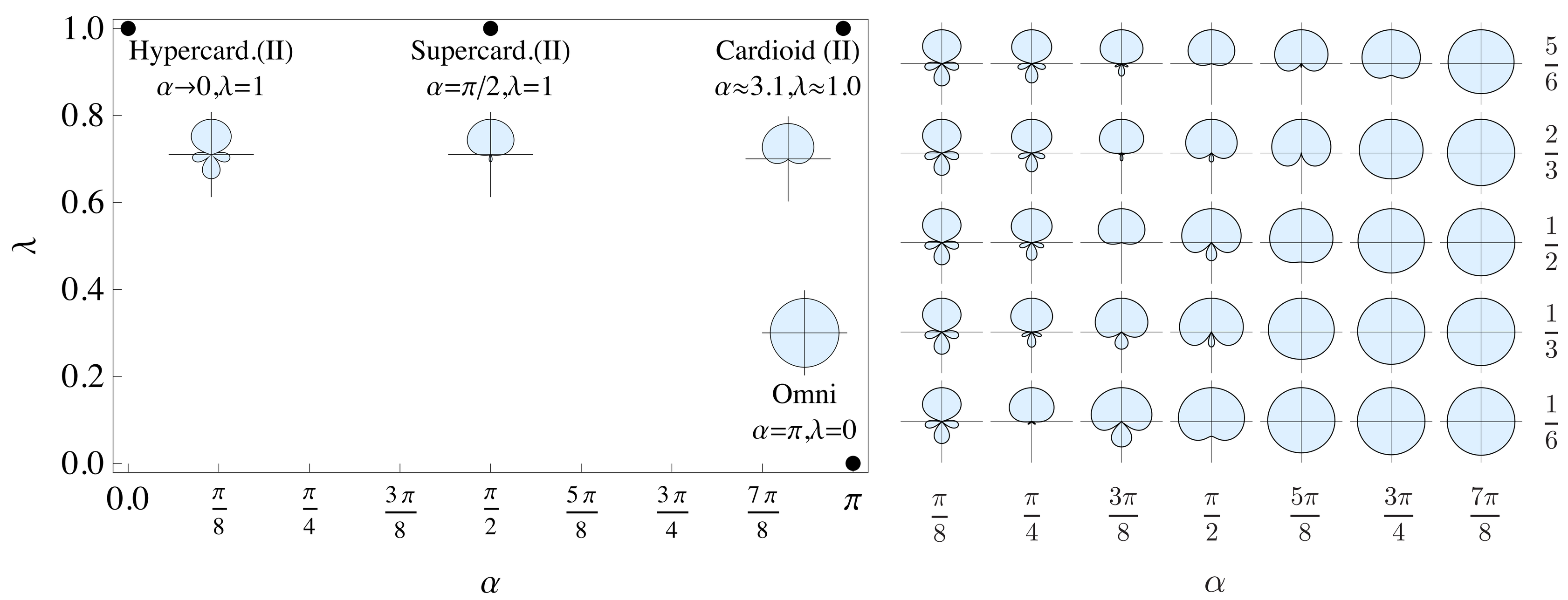
- Subcardioid and cardioid (I-order to III-order): δ lower than -52dB.

- The method provides a front-end for directivity adjustment: set two relevant parameters rather than N coefficients.
- Also provides end-users with standard patterns they might be accustomed to.
- Results and design examples are summarized in the figures.

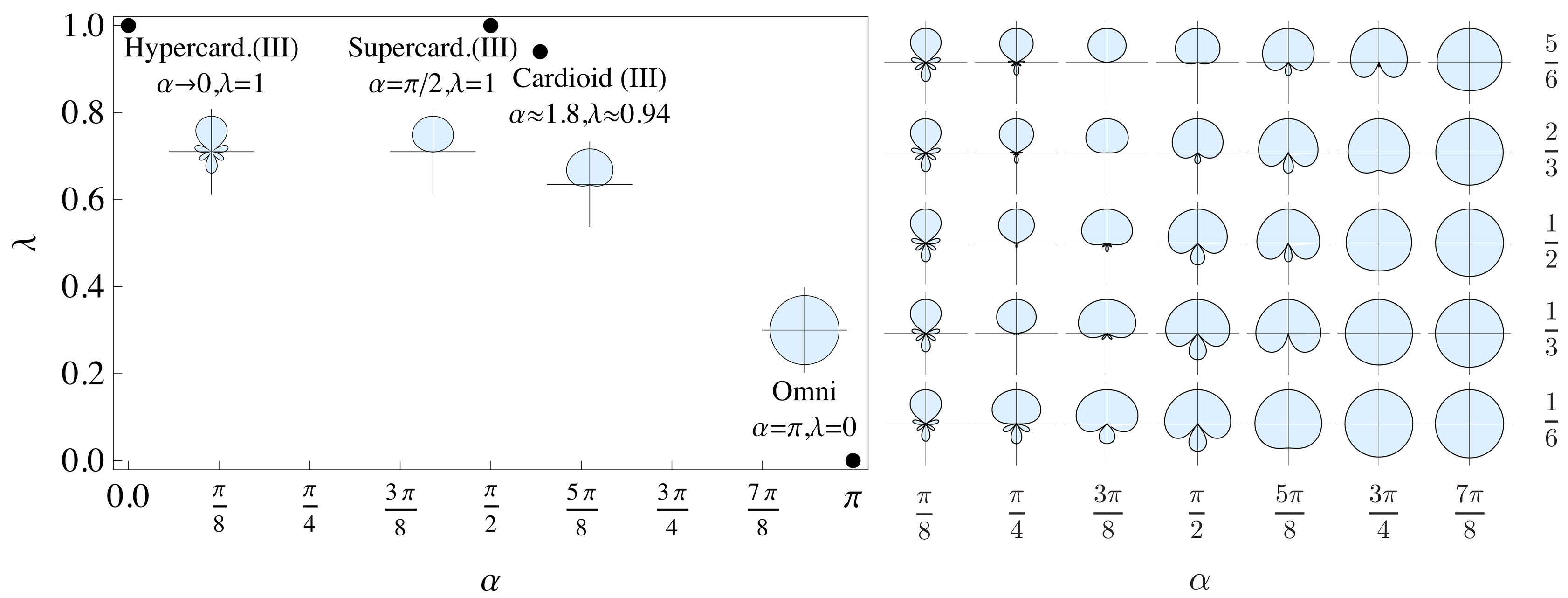
I-ORDER



II-ORDER



III-ORDER



EPSRC
Engineering and Physical Sciences Research Council